Interpretation of space-like solutions of infinite-component wave equations and Grodsky-Streater 'No-Go' theorem

1977 J. Phys. A: Math. Gen. 10 1233

(http://iopscience.iop.org/0305-4470/10/7/020)

J. Phys. A: Math. Gen., Vol. 10, No. 7, 1977. Printed in Great Britain. © 1977

Interpretation of space-like solutions of infinite-component wave equations and Grodsky–Streater 'No-Go' theorem

A O Barut† and J Nagel†
Sektion Physik der Universität München, 8 München 2, Germany

Received 17 May 1976, in final form 1 March 1977

Abstract. The existence of space-like solutions of infinite-component wave equations is not a 'disease' but a virtue. By comparing quantum electrodynamics with the infinite-component wave equations for bound electrons we show that the space-like solutions correspond to relativistic negative-energy solutions of the constituents of the composite system. Hence they have physical consequences in second-order processes such as the Compton effect and electromagnetic polarizabilities. Thus the assumption of the 'No-Go' theorem admitting only time-like solutions is too restrictive for a field theory with infinite-component equations.

1. Introduction

The realistic and successful infinite-component wave equations as applied in the past decade to a relativistic treatment of systems like the H atom or hadrons have, in addition to the positive-mass physical states, also space-like solutions. On the other hand Grodsky and Streater (1968), by requiring positive-mass solutions *only* ('and obviously only time-like values can occur'), found the set of local field theories with infinite-component wave equations to be void. Subsequently attempts were made to construct wave equations without space-like solutions (Bacry and Chang 1973, Nambu 1967, Fronsdal 1968), or to find the physical interpretation of the space-like solutions (Barut 1969, and references therein). Generally, the existence of space-like solutions is seen as a 'disease' and as a result the study of infinite-component wave equations and the related problem of saturation of current algebra relations by one-particle states have received a setback. In this study we show that the space-like solutions have a *definite physical interpretation, they have experimentally demonstrable consequences* and form an integral part of the theory. We do this by comparing quantum electrodynamics of bound states with the corresponding infinite-component wave equations. The space-like solutions are nothing but the negative-energy states of the *bound* electron‡. As the negative-energy states of the *free* electron are re-interpreted in the hole theory as the positron, we have similarly to re-interpret the negative-energy solutions of the *bound* electron. Such states are actually used in problems involving bound electrons, e.g. Compton effect off a bound relativistic electron, or the polarizability of the

† On leave of absence from the University of Colorado, Boulder, Colorado, USA.
‡ Chang and O'Raifeartaigh (1968) point out in this connection that in the free quark model the total momentum of two non-interacting quarks is space-like if one of them has negative energy.

relativistic H atom. Infinite-component wave equations describe composite systems (Takabayashi 1965, Barut and Malin 1968, 1972). Although localizable asymptotic states have positive norm and mass, we show that the negative-normed solutions contribute to the second-order processes in the intermediate states.

The relativistic H atom has been a guide and prototype in the use of infinite-component wave equations for hadrons. Therefore, the present results are also significant for and applicable to the calculation of hadron properties in second-order processes using such equations, such as Compton effect and electromagnetic polarizabilities (Barut and Nagel 1976) where the effect of the negative-energy solutions seems already to be seen experimentally.

2. In quantum electrodynamics

2.1. Free electrons

We begin with a brief review of the situation in quantum electrodynamics. The existence of negative-energy solutions to the relativistic wave equation for the electron was a serious difficulty of the Dirac theory until the discovery of antimatter. When the Dirac field is quantized, the roles of the creation and annihilation operators for the negative-energy solution are interchanged, i.e. it is the absence of a negative-energy state that is associated with the positron. Equivalently, a negative-energy state is regarded as a positive-energy state moving backwards in time. In perturbation theory we must sum over a complete set of solutions in the intermediate states. Hence a second-order process, for example, is at first formally represented by the diagrams of figure 1(a). However, the new interpretation of vacuum as the almost completely filled sea of negative-energy states changes the second diagram of figure 1(a) as shown in figure 1(b).

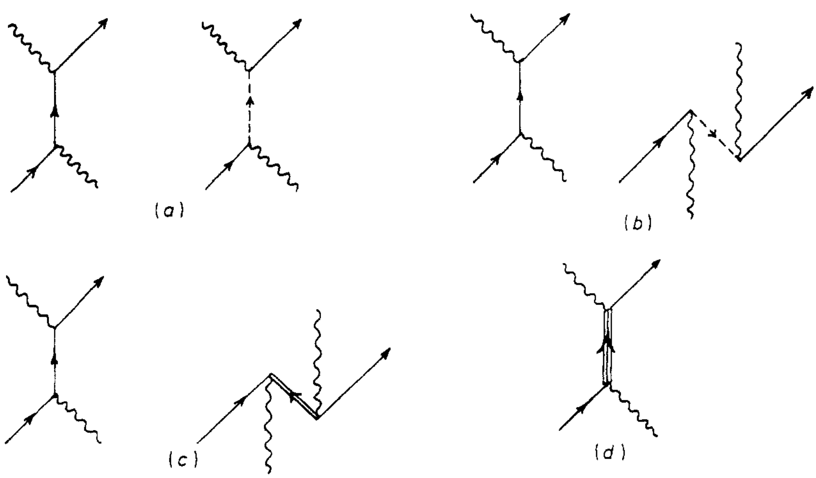

Figure 1. On the interpretation of negative-energy solutions: ——>——, positive-energy electron solutions; - - ->- - -, negative-energy electron solutions; ══>══, positive-energy positron solutions; ≡≡>≡≡, Feynmann propagator. In addition there are crossed photon diagrams in all cases.

Here the second diagram represents the excitation of a negative-energy state, giving a free positive-energy electron and a hole which is later filled by the incoming electron. Equivalently one pictures the negative-energy solution as a positive-energy solution but moving backwards in time. In the quantized Dirac field theory, the second diagram of figure 1(b) is interpreted as the normal creation of an electron–positron pair (figure 1(c)), which is the same in the compact Feynman theory as the simple covariant diagram of figure 1(d).

The propagators corresponding to each part of the figure are:

figure 1(a):

$$\sum_{n^{\pm}} \Psi_n^{(\pm)}(x_1)\bar{\Psi}_n^{(\pm)}(x_2)$$

figure 1(b):

$$\sum_{n^{+}} \Psi_n^{(+)}(x_1)\bar{\Psi}_n^{(+)}(x_2), \qquad t_1 > t_2$$

$$-\sum_{n^{-}} \Psi_n^{(-)}(x_1)\bar{\Psi}_n^{(-)}(x_2), \qquad t_1 < t_2$$

figure 1(c): same as figure 1(b), except the negative-energy solutions are called antimatter solutions:

$$\sum_{n_{e^-}} \Psi_{n_{e^-}}^{e^-}(x_1)\bar{\Psi}_{n_{e^-}}^{e^-}(x_2), \qquad t_1 > t_2$$

$$-\sum_{n_{e^+}} \Psi_{n_{e^+}}^{e^+}(x_1)\bar{\Psi}_{n_{e^+}}^{e^+}(x_2), \qquad t_1 < t_2$$

figure 1(d): for free electrons,

$$(-\mathrm{i}\partial\!\!\!/ + m)\,\Delta(x_1 - x_2;\, m^2).$$

The negative-energy solutions are an integral part of quantum electrodynamics. The Feynman propagator contains implicitly the negative-energy states pictured in figures 1(b) and 1(c).

2.2. Bound electron

We now go over to the composite relativistic systems and consider the bound electron in the case of the Dirac H atom. There are three types of solutions: bound, positive-energy continuum, and negative-energy continuum. The bound states are the usual discrete, localized wavefunctions. The positive-energy continuum states are just ionization states in an attractive potential. The negative-energy states have the same functional form that one obtains for the scattering of two particles of the same charge, i.e. repulsive-potential scattering states. Note that this agrees well with the interpretation of the negative-energy states in terms of antiparticles. If a positron is created in the Coulomb field, it will be Coulomb scattered (figure 2). Thus the negative-energy solutions to the relativistic H problem take into account the process of pair production of one of the constituents, the electron. This is the key to the interpretation of space-like solutions in the case of infinite-component systems, which are composite systems like the H atom.

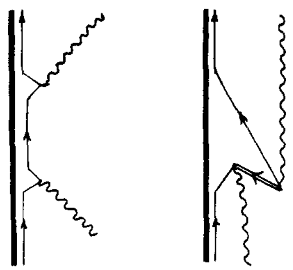

Figure 2. On the interpretation of space-like solutions. (In addition there are crossed photon diagrams.)

2.3. *Effect of negative-energy states in low-energy properties of the system*

In addition to the production of antiparticles, the experimental consequences of negative-energy states can be seen in the low-energy properties of second-order processes.

As a typical second-order process we consider the Compton scattering off a *bound* Dirac particle. The transition amplitude in second order is (as derived from Feynman rules) (Sakurai 1967, Akhiezer and Berestetski 1965):

$$U_{i\to f}=e^2\sum_{n^\pm}\left(\frac{\langle f|\not e_f e^{-i\boldsymbol{k}_f\cdot\boldsymbol{r}}|n\rangle\langle n|\not e_i e^{i\boldsymbol{k}_i\cdot\boldsymbol{r}}|i\rangle}{E_n(1-i\epsilon)-E_i-\omega_i}+\frac{\langle f|\not e_i\, e^{i\boldsymbol{k}_i\cdot\boldsymbol{r}}|n\rangle\langle n|\not e_f\, e^{-i\boldsymbol{k}_f\cdot\boldsymbol{r}}|i\rangle}{E_n(1-i\epsilon)-E_f+\omega_i}\right),\qquad \not e_f=e_f^\mu\gamma_\mu,\ \not e_i=e_i^\mu\gamma_\mu, \tag{1}$$

where use is made of the propagator for a bound electron (figure 2). The sum in (1) is over positive- and negative-energy states. It is interesting to see the effects of the negative-energy intermediate states even in the non-relativistic limit. For $\omega\ll m$ and $\Delta E\ll m$ we can assume $E_n\sim -m$, $E_i\sim m$, $\omega_i\sim 0$ in the denominators of the negative-energy sum. Then (1) becomes:

$$U_{i\to f}=-\frac{e^2}{2m}\sum_{n^-}\left(\langle f|\not e_f\, e^{-i\boldsymbol{k}_f\cdot\boldsymbol{r}}|n\rangle\langle n|\not e_i\, e^{i\boldsymbol{k}_i\cdot\boldsymbol{r}}|i\rangle+\langle f|\not e_i\, e^{i\boldsymbol{k}_i\cdot\boldsymbol{r}}|n\rangle\langle n|\not e_f\, e^{-i\boldsymbol{k}_f\cdot\boldsymbol{r}}|i\rangle\right)$$

$$+\text{sum over positive-energy virtual states.}$$

Using

$$\Psi_n^{(-)}=\frac{m-H}{m-E_n}\Psi_n^{(-)}\cong\frac{m-H}{2m}\Psi_n^{(-)}\qquad\text{and}\qquad\frac{m-H}{2m}\Psi_n^{(+)}\cong 0,$$

we convert the sum over negative-energy states in (2) into a sum over all states, and then use commutation relations of all γ matrices to obtain

$$U_{i\to f}=\frac{e^2}{m}\langle f|\boldsymbol{e}_f\cdot\boldsymbol{e}_i|i\rangle+\text{sum over positive-energy virtual states.}$$

Thus even in the low-energy limit the negative-energy states contribute, giving the so called 'seagull' interaction.

The seagull term arises in all theories with negative energies. This is because if there are negative-energy states in the theory, an external field can create particle–antiparticle pairs, and therefore the electromagnetic current and the particle field itself are not independent of the external field. Since the interaction is of the form $j_\mu A^\mu$, any dependence of j_μ on A_μ leads to a quadratic term in A_μ, giving the seagull term (Jauch and Rohrlich 1955, chap. 14).

For the Dirac H atom we have shown explicitly that the virtual transitions to negative-energy states reduce, in the non-relativistic limit, to the seagull term. It is in fact this term which gives the lowest-order contribution to the magnetic polarizability of the H atom. The result is (Van Vleck 1932)

$$\beta = -\tfrac{1}{2}\alpha^2 a^3,$$

where $a = 1/\alpha m$ is the Bohr radius; β is negative indicating that the negative-energy states give rise to a diamagnetic effect. This is because the virtual production of electron–positron pairs implies a set of charges, and classically the magnetic polarizability of a charge cloud is negative.

It is remarkable that a Hamiltonian linear in the momenta (e.g. Dirac atom) with minimal coupling and negative-energy states, gives rise to the same interaction (i.e. $\boldsymbol{A}^2$), as a quadratic Hamiltonian, but with no negative-energy states.

3. Infinite-component wave equations

The infinite-component wave equations generalize a Schrödinger- or Dirac-type equation *with* a potential into a covariant form although the potential does not appear explicitly and has been eliminated in favour of an algebraic structure. Such equations should therefore be identified with a composite system with internal structure and not with an elementary particle. There is unfortunately no closed, exact passage from the general principles of quantum electrodynamics to the equations describing the bound states. Various approximations, assumptions and postulates have to be made to arrive from the general Bethe–Salpeter framework at a Dirac equation with a Coulomb potential, for example. The infinite-component wave equation is somewhere between the two. It describes the composite system completely covariantly and treats it as an elementary entity with internal structure. It is a dynamical equation derived by an inverse process, from the bound-state equation with potential to a relativistic equation, by using the underlying algebraic structure and spectrum of the problem. There are no problems with consistency and interpretation. However, its disadvantage lies in the fact that the relation of the coefficients of the equation to the masses of the constituents is not known exactly, except approximately in the case of the H atom.

We shall therefore compare the unperturbed infinite-component wave equation to the Dirac equation with potential and develop a systematic perturbation theory when external electromagnetic interactions are included.

3.1. Spectrum

We now consider the infinite-component wave equation describing a bound relativistic particle. The equation is

$$(J_\mu P^\mu + \beta S + \gamma)\Psi(P) = 0, \tag{2}$$

where $J_\mu = \alpha_1\Gamma_\mu + \alpha_2 P_\mu + \alpha_3 S P_\mu$, with α_i, β, γ = constants. Here Γ_μ, S are elements of the O(4, 2) Lie algebra in a specific representation (Jauch and Rohrlich 1955, chap. 14).

The mass spectrum is obtained by diagonalizing (2) in the rest frame:

$$[(\alpha_1\Gamma_0+\alpha_2M+\alpha_3SM)M+\beta S+\gamma]\Psi(0)=0. \tag{2a}$$

We find in general the existence of discrete, continuum, and also space-like states, the 'disease' of infinite-component theories.

The discrete spectrum is given by

$$(M_n^{(\pm)})^2=\frac{\alpha_1^2-2\beta\alpha_3-2\gamma(\alpha_2/n^2)\pm\{\alpha_1^2(\alpha_1^2-4\beta\alpha_3)-(4/n^2)[\alpha_1^2\alpha_2\gamma+(\alpha_2\beta-\alpha_3\gamma)^2]\}^{1/2}}{2(\alpha_3^2+\alpha_2^2/n^2)}, \tag{3}$$

where $n=\frac{3}{2},\frac{5}{2},\frac{7}{2}\ldots$ for fermion representations and $n=1, 2, 3\ldots$ for boson representations.

A careful study of (2*a*) leads to the following lemma.

Lemma. The plus or minus solutions in (3) *both* exist if the condition

$$\alpha_1^2\alpha_2\gamma+(\alpha_2\beta-\alpha_3\gamma)^2>0 \tag{4}$$

holds. In that case the plus sign corresponds to positively-normed, and the negative sign to negatively-normed solutions.

Wave equations for which (4) holds will be denoted *type I*. *Type II* wave equations for which (4) is not true, have either the plus or minus sign in (3), but not both. The norm is then either positive or negative, respectively.

The continuum mass spectrum is given for both types I and II by

$$(M_\lambda^{(\pm)})^2=\frac{\alpha_1^2-2\beta\alpha_3+2\gamma(\alpha_2/\lambda^2)\pm\{\alpha_1^2(\alpha_1^2-4\beta\alpha_3)+(4/\lambda^2)[\alpha_1^2\alpha_2\gamma+(\alpha_2\beta-\alpha_3\gamma)^2]\}^{1/2}}{2(\alpha_3^2-\alpha_2^2/\lambda^2)} \tag{5}$$

where $|\alpha_2/\alpha_3|<|\lambda|<\infty$. The mass spectra for the two types are represented in figure 3. Type I solutions have two sets of discrete states and both the positive and negative continua have the same analytic character as in the case of the ionized H atom wavefunction, which corresponds to scattering of particles of *different* charge.

Type II solutions have only one set of discrete solutions. The upper continuum states again correspond to ionization wavefunctions, but the lower continuum has the same

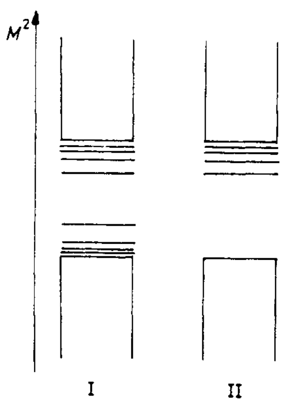

Figure 3. Mass spectra for type I and type II infinite-component wave equations.

analytic character as the scattering wavefunction of two particles of the same charge. We know that the relativistic H atom is well described by an infinite-component wave equation (see the reviews by Barut and Rasmussen 1973).

Considering the boson representation and taking the parameters that correspond to the H atom, we see immediately that the H atom is of type II. For $\alpha^2 \ll 1$ we get the usual discrete spectrum

$$M_n \cong M_{\mathrm{p}} + m_{\mathrm{e}} - (m_{\mathrm{e}}\alpha^2/2n^2). \tag{6}$$

The continuum states have masses in the range

$$(M_{\mathrm{p}}+m_{\mathrm{e}})^2 \leqslant (M_\lambda^{(+)})^2 < \infty \qquad \text{and} \qquad -\infty < (M_\lambda^{(-)})^2 \leqslant (M_{\mathrm{p}}-m_{\mathrm{e}})^2. \tag{7}$$

This situation corresponds exactly to the spectrum in quantum electrodynamics: $(M_\lambda^{(+)})^2$ is the continuum $(\text{mass})^2$, and $(M_\lambda^{(-)})^2$ corresponds to the scattering states of negative energy. The fact that space-like solutions with $(M_\lambda^{(-)})^2 = P_\mu P^\mu < 0$ occur is consequently of no problem here. This is because of the fact that $(p_1+p_2)_\mu = P_\mu$ can be space-like, even though both p_1 and p_2 are time-like. Here we show that the space-like solutions of the infinite-component wave equation for the relativistic H atom are associated with the negative-energy states of the Coulomb problem.

The wave equation for the proton, in order to fit the observed form factors, must be of type I. This implies a different kind of constituent particle than in the H atom, which is not surprising. The main difference is that we have the existence of negative-norm discrete states—in the H atom this would have corresponded, if they existed at all, to a positron bound to a proton! The proton constituents must be of a strange form; for example, in the dyonium model they are magnetic charges in parity eigenstates (Barut 1971):

$$|g\rangle \pm |-g\rangle.$$

The interpretation of the space-like solutions of the proton is the same as before: production of particle–antiparticle pairs of constituents. The fact that no constituents have ever been seen is understood if the constituents interact so strongly that, at all energies so far obtainable, the produced pair immediately interact to form a pion. This idea is supported by the O(4, 2) spectra for the proton. Equation (5) gives $M \cong 0{\cdot}8$ GeV for the highest space-like state of the proton, giving a threshold for importance of space-like states of $0{\cdot}94 - 0{\cdot}8 = 0{\cdot}14$ GeV, which is precisely the pion mass. Thus the effects of space-like states of the proton are to be seen in pion production.

We have seen, for the Coulomb problem, that the negative-energy states contribute to low-energy characteristics such as the Thomson term and polarizabilities. In the infinite-component theory for the relativistic H atom, these same negative-energy states correspond to the space-like solutions. Thus we expect space-like states in infinite-component field theories to be physically evident in terms of low-energy properties.

3.2. The role of space-like solutions

To obtain the Compton amplitude for a particle in O(4, 2) theory, we first obtain the interaction terms by minimal coupling with the infinite-component wave equation (3). The result is:

$$eJ_\mu A^\mu + e^2(\alpha_2 + \alpha_3 S)A_\mu A^\mu,$$

where the second term is the seagull interaction. The Compton amplitude is then:

$$T_{\mu\nu} = e^2\langle 0|J_\mu \Omega^{-1} J_\nu|0\rangle + e^2\langle 0|J_\nu \Omega^{-1} J_\mu|0\rangle + e^2\langle 0|(\alpha_2+\alpha_3 S)|0\rangle g_{\mu\nu}, \qquad (8)$$

where

$$\Omega = J_\mu P^\mu + \beta S + \gamma,$$

$$J_\mu = \alpha_1 \Gamma_\mu + 2\alpha_2 P_\mu + 2\alpha_3 S P_\mu + \mathrm{i}\alpha_4 L_{\mu\nu} q^\nu + \tfrac{1}{2}\mathrm{i}\alpha_5 \epsilon_{\mu\nu\sigma\rho} L^{\sigma\rho} q^\nu.$$

To get the Thomson term from this, we use $P_\mu = (M, 0, 0, 0)$.

Then it can easily be verified using the commutation relations of the Lie algebra that (Barut and Rasmussen 1973):

$$\begin{aligned} &[M_i, \Omega] = \mathrm{i}\alpha_1 m \Gamma_i \\ &[M_i, \Gamma_j] = \mathrm{i}\delta_{ij}\Gamma_0 \\ &[\boldsymbol{M}\cdot\boldsymbol{e}_\mathrm{i}, \alpha_1 \boldsymbol{\Gamma}\cdot\boldsymbol{e}_\mathrm{f}] = \mathrm{i}\boldsymbol{e}_\mathrm{i}\cdot\boldsymbol{e}_\mathrm{f}[J_0 - (\alpha_2+\alpha_3 S)m]. \end{aligned} \qquad (9)$$

So we have the result

$$\langle 0|(\alpha_2+\alpha_3 S)|0\rangle = \frac{1}{m}\langle 0|J_0|0\rangle + \frac{\mathrm{i}}{m}\langle 0|[\boldsymbol{M}\cdot\boldsymbol{e}_\mathrm{i}, \alpha_1\boldsymbol{\Gamma}\cdot\hat{\boldsymbol{e}}_\mathrm{f}]|0\rangle.$$

Now

$$T = e_i T_{ij} e_j = e^2\left\langle 0\left|\frac{-\mathrm{i}}{m}[M_i, \Omega]\Omega^{-1}\alpha_1\Gamma_j\right|0\right\rangle e_i e_j + e^2\left\langle 0\left|-\alpha_1\Gamma_j\Omega^{-1}\frac{\mathrm{i}}{m}[M_i, \Omega]\right|0\right\rangle e_i e_j$$

$$+\frac{e^2}{m}\langle 0|J_0|0\rangle\hat{\boldsymbol{e}}\cdot\hat{\boldsymbol{e}} + \frac{\mathrm{i}}{m}\langle 0|[\boldsymbol{M}\cdot\hat{\boldsymbol{e}}, \alpha_1\boldsymbol{\Gamma}\cdot\hat{\boldsymbol{e}}]|0\rangle.$$

Using the fact that $\Omega|0\rangle = \langle 0|\Omega = 0$, we easily obtain

$$T = \frac{e^2}{m}\langle 0|J_0|0\rangle\hat{\boldsymbol{e}}\cdot\hat{\boldsymbol{e}},$$

which is the Thomson term. Note that the seagull term $(\alpha_2+\alpha_3 S)A_\mu A^\mu$ is vital to this conclusion. This interaction is intimately related to the existence of space-like solutions. When α_2 and α_3 are zero, the space-like states disappear and the mass spectrum is then decreasing with spin instead of increasing.

To obtain the polarizabilities we have used the interaction terms (7) and formulated the perturbation theory in the space of group functions (Barut and Nagel 1977). This method is similar to the calculation of non-relativistic H atom polarizabilities in parabolic coordinates. The simplification is that only one or two virtual states need to be summed over rather than the entire discrete and continuous spectra. The results give $\alpha > \beta$ in agreement with experiment, whereas we might have expected $\beta > \alpha$ due to the magnetic character of the (33) resonance. The reason we obtain a smaller magnetic polarizability can be traced to the diamagnetic contribution from transitions to space-like states.

To summarize, the space-like solutions of infinite-component field theory are physical and have experimental consequences. They are intimately connected with the compositeness of the particle. For further discussions on the relationship of infinite-component wave equations to field theory, see Brézin *et al* (1970), Itzykson *et al*

(1970), Todorov (1970), Fronsdal and Huff (1971), Fronsdal and Lundberg (1970) and Barut and Baiquni (1969).

3.3. *Discussion: transitions to space-like states*

It must be emphasized that we are dealing with the state-space of a composite system, and not with that of a standard local quantum field theory. Moreover, this space is much more than the intuitive bound- and scattering-states of non-relativistic composite systems. In the relativistic case, we must also allow additional states of the system corresponding to the negative-energy states of one or more of the constituents. Are we therefore really justified to exclude such states *a priori* as does the axiom system of Streater and Grodsky? The basis of the axiom allowing only time-like states is the intuitive picture that one detects in a scattering experiment asymptotically definite (particle) states which are in principle localizable. The localization of relativistic time-like states is relatively straightforward. An irreducible representation of the Poincaré group with $M^2 > 0$ when restricted to the Euclidian group gives a representation of the latter which is induced from the spin group SU(2) (Newton and Wigner 1949, Wightman 1962). Hence definite localized states have the intuitive invariance property under spatial rotations and translations. For light-like states the localization is more complicated, and it becomes even more so for space-like states (Barut 1976). Even the detection of a system consisting of two particles, one of which is in a negative-energy state, clearly requires a different *space–time* arrangement for the detector than for a time-like state.

Therefore, we cannot *a priori*, both on physical and mathematical grounds, exclude the space-like states from the picture even for asymptotic states, and there might indeed be transitions to space-like states which could be calculated and compared to experiments if one could detect space-like states asymptotically.

There is, however, in relativistic quantum theory an additional principle of re-interpretation of negative-energy states, in two forms; either the hole theory of Dirac, or the second quantization formalism, as discussed in § 2. Now, if all the negative-energy states for a constituent are filled, then we have to conclude that the space-like states of the infinite-component wave equation are also completely filled. In the intermediate states there will be virtual transitions to space-like states if there are holes created in the negative-energy states of a constituent by the ejection of a particle, as in the example of figure 2. In the language of second quantization, we must re-interpret the space-like states of a bound electron, for example, as the time-like states of a bound positron. Thus, the problem of transition to space-like states is on the same footing as the transition to negative-energy states in ordinary quantum electrodynamics, and is no more mysterious or fearsome.

We believe the physical interpretation given here to be important, because now the way is open for a formal second quantization procedure for infinite-component wave equations which is still an open problem.

Finally, one should answer the question: Why do we need a relativistic theory of composite systems, if we have a local field theory in terms of the constituents? The main reason is that the local field theory does not give us immediately the non-perturbative bound- and resonance-states and their intrinsic properties which are the essence of the infinite-component wave equations. At a time when one speculates about unobservable or eternally confined constituents, such as quarks, a theory dealing with the system as a whole and not with the constituents, is clearly very desirable.

Acknowledgment

We want to thank Dr J Kraus for many interesting discussions.

References

Akhiezer A I and Berestetski V B 1965 *Quantum Electrodynamics* (New York: Wiley, Interscience)
Bacry H and Chang N P 1973 *Phys. Lett.* **44B** 286
Barut A O 1969 *Lett. Nuovo Cim.* **1** 601
—— 1971 *Phys. Rev.* D **3** 1747
—— 1976 *Proc. Conf. on Tachyons and Related Problems, Erice* to be published
Barut A O and Baiquni A 1969 *Phys. Rev.* **184** 1342
Barut A O and Malin S 1968 *Nuovo Cim.* A **58** 835
—— 1972 *Ann. Phys., NY* **69** 463
Barut A O and Nagel J 1976 *Phys. Rev.* D **13** 2075
—— 1977 *J. Phys. A: Math. Gen.* **10** 1243
Barut A O and Rasmussen W 1973 *J. Phys. B: Atom. Molec. Phys.* **6** 1695, 1713.
Brézin E, Itzykson C and Zinn-Justin J 1970 *Phys. Rev.* D **1** 2349
Chang S J and O'Raifeartaigh L 1968 *Phys. Rev.* **170** 1316
Fronsdal C 1968 *Phys. Rev.* **171** 1811
Fronsdal C and Huff R 1971 *Phys. Rev.* D **3** 933
Fronsdal C and Lundberg L E 1970 *Phys. Rev.* D **1** 3247
Grodsky L T and Streater R F 1968 *Phys. Rev. Lett.* **20** 695
Itzykson C, Kadyshewskii V G and Todorov I T 1970 *Phys. Rev.* D **1** 2823
Jauch J M and Rohrlich 1955 *Theory of Photons and Electrons* (New York: Addison-Wesley)
Nambu Y 1967 *Phys. Rev.* **160** 1171
Newton T D and Wigner E P 1949 *Rev. Mod. Phys.* **21** 400
Sakurai J J 1967 *Advanced Quantum Mechanics* (New York: Addison-Wesley)
Takabayashi T 1965 *Prog. Theor. Phys.* **34** 124
Todorov I T 1970 *Lecture Notes in Physics* vol. 6 (*Battelle Rencontres* 1969) (Berlin: Springer)
Van Vleck J H 1932 *Theory of Susceptibilities* (London: Oxford University Press)
Wightman A S 1962 *Rev. Mod. Phys.* **34** 845